

# Determining bottom price-levels after a speculative peak

B.M. Roehner<sup>a</sup>

LPTHE, University Paris 7, 2 place Jussieu, 75005 Paris, France

Received 9 June 2000

**Abstract.** During a stock market peak the price of a given stock ( $i$ ) jumps from an initial level  $p_1(i)$  to a peak level  $p_2(i)$  before falling back to a bottom level  $p_3(i)$ . The ratios  $A(i) = p_2(i)/p_1(i)$  and  $B(i) = p_3(i)/p_1(i)$  are referred to as the peak- and bottom-amplitude respectively. The paper shows that for a sample of stocks there is a linear relationship between  $A(i)$  and  $B(i)$  of the form:  $B = 0.4A + b$ . In words, this means that the higher the price of a stock climbs during a bull market the better it resists during the subsequent bear market. That rule, which we call the resilience pattern, also applies to other speculative markets. It provides a useful guiding line for Monte Carlo simulations.

**PACS.** 64.60.Fr Equilibrium properties near critical points, critical exponents – 87.23.Ge Dynamics of social systems

## 1 Introduction

Traffic jams are fairly unpredictable because they depend upon a large number of factors (*e.g.* timing in the traffic, weather conditions, highway maintenance, automobile accidents), some of which are completely random. However once begun, traffic jams display fairly recurrent patterns as to average duration, behavior of the drivers, and so on. This traffic jam parallel has been introduced by Charles Tilly [12] in the context of historical sociology in order to explain why revolutions can hardly be predicted. It also applies to the occurrence of speculative price peaks: the downturn of price peaks can hardly be predicted because they depend upon a number of (possibly) exogenous factors<sup>1</sup>. However, as is the case for traffic jams, once a price peak is under way it obeys some definite rules; accordingly its outcome can to some extent be predicted at the level of individual companies.

More precisely we focus our attention on the relationship between the prices at the beginning of the peak, at the peak and at the end of the peak. We denote by  $p_1$  the price level at the start of the peak, by  $p_2$  the price at the peak and by  $p_3$  the bottom price at the end of the falling price path; we further introduce the peak

amplitude  $A = p_2/p_1$  and the bottom amplitude  $B = p_3/p_1$ .  $A$  and  $B$  can be defined for any company in the market; for instance on the NASDAQ where there are currently more than 5000 companies listed  $A$  and  $B$  can be seen as variables for which there are several thousand realizations. In the next section we show that  $A$  and  $B$  are closely correlated; with a correlation of the order of 0.75 the following regression holds:

$$B = aA + b \quad (1)$$

where  $a$  is usually of the order of 0.4. The fact that  $a$  is positive means that the higher the peak amplitude, the larger the bottom amplitude; in other words the higher the price of a stock climbs during the rising phase (bull market) the better it resists during the falling phase (bear market). The regularity summarized by equation (1) will be referred to as the resilience pattern. The paper proceeds as follows. In the second section the statistical methodology is explained, then the resilience pattern is established and illustrated through several case studies: we consider three stock market peaks, one price bubble for postage stamps and two real estate bubbles. In the conclusion we discuss the possible implications of the resilience pattern.

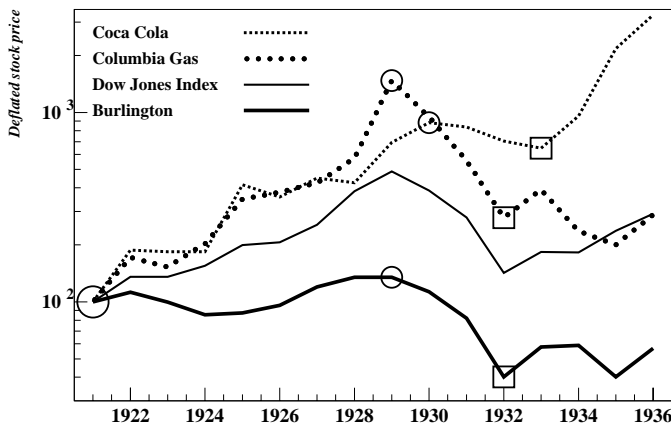
## 2 The resilience pattern

### 2.1 Methodology

Let us consider a typical price peak such as the one in Figure 1. While it is easy to identify the moment  $t_2$  when the price reaches its peak level  $p_2$ , the determination of the initial moment  $t_1$  (corresponding to  $p_1$ ) and the end moment  $t_3$  (corresponding to  $p_3$ ) is not so easy. Fortunately,

<sup>a</sup> e-mail: roehner@lpthe.jussieu.fr

<sup>1</sup> For instance it has been argued (Business Line 8 May 2000) that the spurt in the NASDAQ composite index that occurred in December 1999 and January 2000 was fueled by a Y2K-motivated injection of money into the banking system. Needless to say, no model will ever be able to take into account such circumstantial factors. However, the present paper suggests that when a market rallies or plummets there are some structural invariants which are independent of triggering factors.

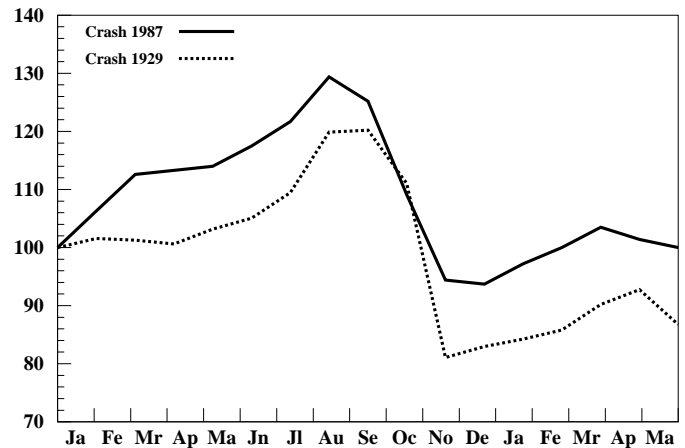


**Fig. 1.** Course of the price for three stocks *versus* Dow Jones index. The prices are yearly highs. During the bull market 1921-1929, the prices of Coca Cola and Columbia Gas (an utility company) have increased faster and more than the DJ average, while the price of Burlington Northern (a railroad company) has increased slower and less than the DJ. The circles and squares show the peaks and troughs respectively. The resilience pattern reveals itself in the fact that when a peak is above (or below) the DJ average the same situation prevails for the trough. Sources: Common stock price histories (log supplement); Dow Jones Investor's handbook (1972).

as will be seen subsequently, the relationship (1) does not depend upon the choice of  $t_1$  or  $t_2$  in a critical way. A peak will be delimited in two steps (i) At the global level of the whole market the price peak is identified by using a broad annual index; in that way one already gets a rough definition of  $t_1, t_2, t_3$  (ii) At the level of individual companies  $t_1$  will be selected as the first year for which the annual price change is positive,  $t_2$  as the peak year and  $t_3$  as the last year for which the annual price change is negative. Let us illustrate the procedure on an example (Fig. 1). As one knows there was a short bull market on the NYSE after World War I which culminated in 1920; thus, for the great majority of the stocks the first positive price change was 1921-1922, which leads us to  $t_1 = 1921$ . At the end of 1929 the downturn occurred very abruptly which means that for almost all stocks  $t_2 = 1929$ . By and large the market bottomed out in 1931; yet for some stocks such as for instance Consolidated Edison the fall continued until 1935; in that case one would take  $t_3 = 1935$ . Once the limits of the peak have been determined for each individual stock, the ratios  $p_2/p_1$  and  $p_3/p_1$  are computed for the deflated prices; then the correlation and regressions are carried out for the sample of stocks under consideration. We now apply this procedure to several case studies.

## 2.2 Stock markets

As far as stock markets are concerned there is often a tendency to over-emphasize the importance of crashes, by which we understand a rapid fall occurring within one or two weeks, at the expense of the long and steady declines that (in some cases) follow the crash. Crashes



**Fig. 2.** Parallel between the crashes of 1929 and 1987 on the NYSE. Monthly prices. During the 8 months before and after the crash the behavior of the Dow Jones index was very much the same in both cases; as one knows the subsequent evolution was very different however. This suggests that crashes and long-lasting slides are two distinct (and not necessarily related) phenomena. Sources: The Dow Jones averages 1885-1970; OECD main economic indicators (1969-1988).

are impressive because of their suddenness, but it is not obvious and probably not even true, that crashes have a determining influence on the medium-term (*i.e.* yearly) evolution of markets. A slide that continues for over two or three years may have more significance for the stock market and for the rest of the economy than the crash itself. A case in point is the parallel between the crashes of October 1929 and October 1987 on the NYSE (Fig. 2). The price paths were very similar during the crashes and the six subsequent months; but, as one knows, the ultimate outcomes were very different. This observation suggests that steady slides (which are the topic of this paper) and abrupt crashes are two different phenomena.

Applying the procedure delineated above, we obtain the results given in Table 1. Note that the downturn in Paris occurred in February 1929 and cannot therefore be considered as a consequence of the Wall Street crash; incidentally, in Germany the downturn took place in June 1928 that is to say more than one year before the downturn on the NYSE. The analysis for the NYSE is based on the behavior of 85 individual stocks. The correlation is 0.87 for equation (1) and the distribution of sample points in the  $(A, B)$  plane is shown in Figure 3; it displays the range of both amplitudes and permits to verify that there is no non-linear effect.

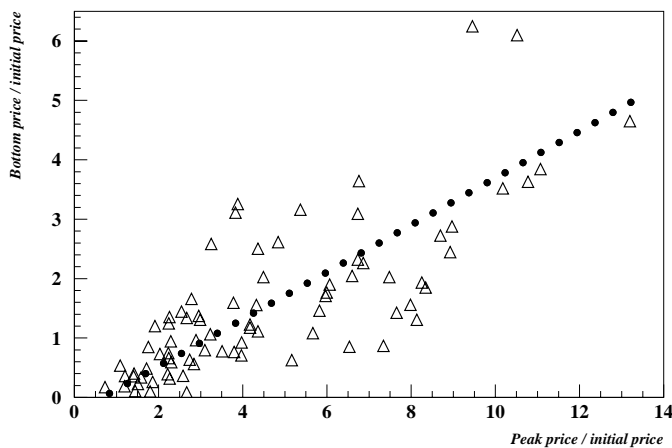
Since this is the largest sample considered in Table 1, it can be of interest to take a closer look at the statistical distribution of the amplitudes:  $A$  and  $B$  have an average of 5.6 and 2.0 respectively and the standard deviations are 5.0 and 2.3. Moreover it turns out that both  $A$  and  $B$  are distributed according to a log-normal density. This could have been expected; indeed, stock prices follow a log-normal law, at least in first approximation that is to say for time-samples of moderate size and time intervals larger than one day, and one knows that the ratio of two

**Table 1.** The resilience pattern: relationship between peak amplitude ( $A$ ) and bottom amplitude ( $B$ ):  $B = aA + b$ .

	Market	Peak	Number of items	$a$	$b$	Correlation
Stocks						
1	NYSE	1929 Oct	85	$0.40 \pm 0.05$	$-0.27 \pm 0.24$	0.87
2	Paris	1929 Feb	19	$0.44 \pm 0.17$	$-0.03 \pm 0.29$	0.76
3a	Tokyo	1989 Oct	26	$0.39 \pm 0.11$	$0.27 \pm 0.10$	0.80
3b	Tokyo	1989 Oct	26	$0.40 \pm 0.09$	$0.35 \pm 0.13$	0.87
Stamps						
4	France	1944	56	$0.10 \pm 0.08$	$0.77 \pm 0.10$	0.33
Real estate						
5	Paris	1990	20	$0.43 \pm 0.16$	$0.73 \pm 0.04$	0.79
6	Paris	1990	5	$0.14 \pm 0.26$	$1.1 \pm 0.1$	0.52
7	Britain	1989	11	$-0.10 \pm 0.17$	$1.4 \pm 0.05$	-0.38
Average (except 7)				0.32		0.68

Notes: All prices used in the regression are real (*i.e.* deflated) prices. Case (3a) refers to  $t_1 = 1985$ , while case (3b) refers to  $t_1 = 1980$ . For some reason yet to be understood the housing bubble in Britain does not follow the resilience pattern (negative correlation).

Sources: 1: [1]; 2: *Annuaire Statistique de la France, Résumé Rétrospectif (1966, p.541)*; 3a,b: *Japan Statistical Yearbook (various years)*; 4: [5]; 5,6: *Conseil par des notaires (23 Dec. 1991) and Chambre des Notaires*; 7: *Halifax index*.



**Fig. 3.** NYSE, 1921-1932 peak: distribution of 85 sample points in the ( $A, B$ ) plane. Each triangle corresponds to a stock; the prices are yearly highs. Dotted line: linear regression (correlation is 0.87).

log-normal random variables is also a log-normal random variable.

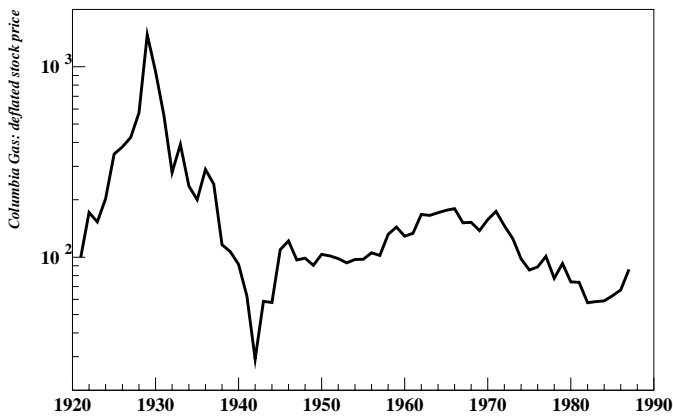
The analysis of the Paris stock market is not based on individual stocks but on a set of indexes corresponding to 19 different economic sectors, *e.g.* banks, coal mines, railroads, electricity, chemicals, etc. Some indexes comprise more individual stocks than others, for instance the bank index comprises 20 banks while the electricity index has only 11; on average there are about 14 companies per sector. The results for the  $B$  versus  $A$  regression are given in Table 1: the  $a$  estimate is fairly close to the one obtained for the NYSE. The analysis of the 1989 peak on the Tokyo market is also based on indexes

corresponding to different economic sectors. With as many as 26 different sectors the classification given in the Japan Statistical Yearbook is even more detailed than the previous one. It is not obvious whether that peak began in 1985 or in 1980. It is true that the increase between 1980 and 1985 was not monotonic, but one can argue with good reason that these fluctuations were rather circumstantial. It is reassuring to observe that by taking  $t_1 = 1980$  one is lead to estimates which are fairly similar to those obtained for  $t_1 = 1985$ ; the fact that the correlation is higher in the first case in fact suggests that  $t_1 = 1980$  is the most “natural” starting point.

On the basis of the resilience pattern it could seem that one can invest in high-growth companies without much risk. This is not completely true however for that rule only concerns the medium-term behavior in the vicinity of a given peak; it does not guarantee that in the long-run the price of a stock which has experienced a huge peak will continue to increase. A spectacular counter-example is shown in Figure 4. Not only did the price of Columbia Gas System never again reach the level it had attained in 1929 but it remained far below.

### 2.3 Other speculative markets

The consideration of other speculative markets relies on the implicit assumption that the *basic* mechanisms of speculation are similar for any speculative market. In this paragraph we examine the cases of postage stamp and real estate markets. These markets are particularly suited for this kind of investigation because (i) they have large price peaks and (ii) they comprise a large number

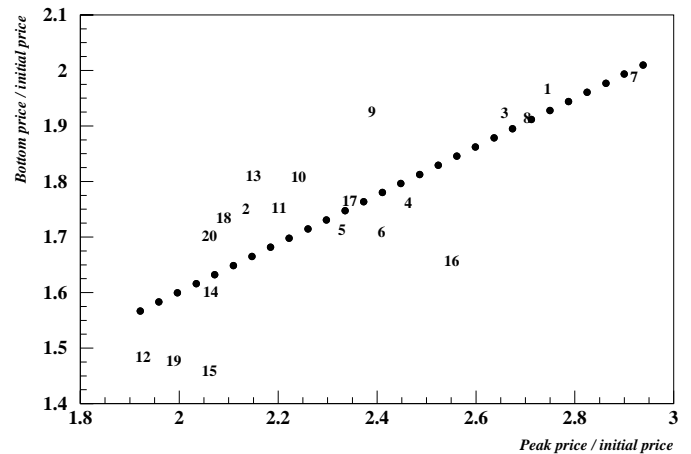


**Fig. 4.** Long-term behavior of the price of Columbia Gas System. The prices are yearly highs. The price of that utility stock increased tremendously during the 1921-1929 bull market; then, from 1932 to 1936 it fluctuated around 200 before falling to about 30; it never really recovered in the second half of the 20th century. The chart does not cover the 1990s because there was a change in the name of the company; however it can be noted that during the 1990-2000 bull market the Dow Jones Utility index increased 1.7 times less than the Standard and Poor's 500; this would give a stock price of about 200 in year 2000, still well below the 1929 high. Source: Common stock price histories (log supplement).

of different items which will play the same role as individual stocks or economic sectors in our previous study.

During World War II there was a postage stamp bubble in France which was triggered by the fact that inflation was high (the consumer price index increased from 130 in 1941 to 350 in 1945) while at the same time, due to war restrictions there was only a small outlet for consumption. Thus almost all stamps experienced a price peak (usually culminating in 1944) with their real (*i.e.* deflated) price multiplied by a factor of 3 or 4. The present investigation focuses on 19th century French stamps. Table 1 shows that the correlation between peak and bottom amplitude although still significant is smaller than in previous cases. This can possibly be attributed to the fact that the prices given in stamp catalogs are estimates made by stamp experts and traders rather than real market prices. Estimates are particularly difficult to make for stamps which are not traded very often; such is the case for the stamps which are particularly rare and expensive. Now (see in this respect [11]) the expensive stamps are also those for which the peak amplitude  $A$  is largest and any bias for large  $A$  sample points will notably affect the correlation and the slope of the regression line.

Our next example concerns the real estate market in Paris. Between 1985 and 1995 there was a price peak which resulted in a doubling, and in some areas a three fold increase, of apartment prices. First, we consider the prices in 20 different districts ("arrondissements") of downtown Paris (*i.e.* the so-called "Paris intra-muros" area). The regression leads to values for the correlation and for the slope of the regression line which are similar to those obtained in the case of stock markets. As shown in



**Fig. 5.** Paris, 1985-1995 real estate peak: distribution of 20 sample points in the  $(A, B)$  plane. The numbers correspond to the 20 arrondissements. Source: Chambre des Notaires.

Figure 5 the sample points in the  $(A, B)$  plane are evenly distributed along the regression line and do not display any obvious non-linear effect.

The second result concerns the same price peak but for apartments according to their size (from one- to five-room). Not surprisingly, due to the small number of sample points the error margin is fairly large; note that the value  $a = 0.4$  obtained in previous cases is within the error bars.

Up to that point all our results were consistent with the resilience pattern, but the following case lead to an unexpected result. It concerns the real estate bubble which occurred in Britain in the 1980s; from the area of London where it started, it spread progressively northward to the rest of the country. Price results can be analyzed at the level of each of the 11 regions composing Britain. The result came as a surprise; in this case the amplitudes are negatively correlated and the correlation is fairly low. In order to see if the negative correlation is found elsewhere it would be of great interest to perform a similar test for other countries; unfortunately, in France reliable regional housing prices are only available since 1995.

### 3 Conclusion

About 60 percent of the 222 sample points in Table 1 concerned stock markets. Thus, the evidence supporting the resilience pattern for stock markets was particularly strong. Yet, it was important also to show that the resilience effect is *not* confined to stock markets for it suggests that a possible theoretical framework should apply to other speculative markets as well. Let us now briefly discuss the significance and possible implications of the present finding. In the late 1990s econophysicists along with some economists ([3]) devoted great attention to the statistical analysis of stock market indexes, the overall objective being the identification of possible scaling laws. Yet, indexes do not give great insight into the internal

mechanisms of stock markets. Such an understanding can only be gained by opening the “black box” and studying the interactions that take place between individual stocks. This idea has recently gained more acceptance as shown by a number of innovative papers going into that direction; *e.g.* [3,4,6]. Finally, let us briefly consider the next step, namely the construction of a theoretical framework. Obviously any model is (and has to be) a schematization of the real world; therefore, constructing a “realistic” model cannot be a viable and suitable objective; models need more precise “targets” and “guiding lights”. In a number of recent empirical studies we have tried to define such targets: the sharp peak - flat trough pattern [9,10], the price multiplier effect [7,9], the relationship between stock market crashes and increases in interest rate spread [8] define quantitative patterns which provide useful guiding lights for the construction of a theoretical framework. On the theoretical side some promising advances have been made recently which can possibly provide an adequate framework for the description of the internal machinery of stock markets. For instance one would not be surprised to see percolation (see in this respect [2]) play a role in the spread of a bubble; after all, a speculative outburst can be seen as propagating from high-growth stocks to low-growth stocks in the same way as a technical innovation progressively gains acceptance.

I would like to express my gratitude to the statistical experts of the Halifax Company (UK) and the Chambre des Notaires (Paris) for their kind assistance.

## References

1. *Common Stock price histories 1910-1986 and logarithmic supplement* (WIT Financial Publishers, Anchorage, 1988).
2. J. Goldenberg, B. Libai, S. Solomon, N. Jan, D. Stauffer, *Physica A* (to appear, 2000); **cond-mat/0005426**.
3. T. Lux, *Applied Financial Economics* **6**, 463 (1996).
4. R.N. Mantegna, *Eur. Phys. J. B* **11**, 193 (1999).
5. A. Massacrier, *Prix des timbres-poste classiques de 1904 à 1975*, edited by A. Maury (Paris, 1978).
6. V. Plerou, P. Gopikrishan, L.A.N. Amaral, M. Meyer, H.E. Stanley, *Phys. Rev. E* **60**, 6519 (1999).
7. B.M. Roehner, *Eur. Phys. J. B* **14**, 395 (2000).
8. B.M. Roehner, *Int. J. Mod. Phys.* **11**, 91 (2000).
9. B.M. Roehner, *Hidden collective factors in speculative trading* (to appear) (Springer-Verlag, Berlin, 2001).
10. B.M. Roehner, D. Sornette, *Eur. Phys. J. B* **4**, 387 (1998).
11. B.M. Roehner, D. Sornette, *Int. J. Mod. Phys. C* **10**, 1099 (1999).
12. C. Tilly, *European revolutions 1492-1992* (Blackwell, Oxford, 1993).